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Definition and examples

To give the images a normal perspective, the artists began creating their work by drawing the horizon line $h$ at the level of the eye. Then at the center of the line they marked the main point $A$, which is the point of intersection of all the parallel lines. At a distance equal to the distance of the eye to a central point, there was point $B$ marked there. This is a point of the intersection of diagonals. To be read properly, the image should be viewed from a particular viewpoint. In this way a square of the grid on the ground after projecting appears to be a trapezoidal checkerboard, on which it is enough to put other items in proportion to the decreasing dimensions of the boxes.
Definition and examples

Anamorphosis is like a deviation from the norm. Although the word first appears in the seventeenth century, but is referring to the images known much earlier.

In summary anamorphosis (Gr. ana - back, morphe-form) is an extreme consequence of linear perspective, which involves the deformation of the image by placing the vanishing point of the pyramid of vision away from the main point and observation point close to the plane of the work.

The idea of anamorphosis appeared as a byproduct on the investigation of oblique images and wide-angle views by Piero della Francesca and Leonardo da Vinci.
Construction of perspectival anamorphosis with deformation grids

In the $\mathbb{R}^3$ space we have a plane $\mathfrak{a}$ (called wall) and a vertical plane $\mathfrak{b}$, perpendicular to $\mathfrak{a}$. Let us assume that we are given an eye point $O$, and also a squares grid in a plane $\mathfrak{b}$. This grid will be projected from $O$ upon a wall $\mathfrak{a}$. At the beginning we consider the images of the horizontal lines of the grid. The orthogonal projection of $O$ upon the wall gives us the point $O'$ which is the vanishing point of the set of parallel lines. Thus the images of the horizontal lines are lines radiating out from $O'$. To project the vertical lines upon the wall we use the diagonal method. So we intend to construct the images of the diagonal $AB$ of the grid. This image of this line on the wall intersects the deformation grids in the point $B$. Its vanishing point is point $P$ which we obtain at the intersection of the line passing $O$ and parallel to $AB$. The angle $\angle P O' B = 45^\circ$ so we construct the point $P$ on the vertical line through $O'$ in such a way that $PO'=O'O$. Due to one of the most fundamental theorems of the perspective theory $PB$ is an image of $AB$. The images of vertical lines of the grid we obtain as vertical lines at the point of intersection of images of horizontal lines with lines $PB$. 

Definition and examples - R. Paprocki- Niedzica
**Descriptive geometry**

1. We wish to represent three-dimensional figures in a plane.
2. We choose two mutually perpendicular planes: one is called the ground (or horizontal) plane \( a \) while the other is called the vertical plane \( b \).
3. Their intersection is the ground line \( t \).
4. We project a point \( A \) in a space orthogonally on those two planes.
5. This gives \( A' \) (in \( b \)) and \( A'' \) (in \( a \)).

**A cone and a cylinder in Monge projection**

This figures represent a cone and a cylinder.

**Definition and examples – conical anamorphosis**

**Conical anamorphosis**

1. Imagine a right circular cone mirror standing on the ground plane and an eye point \( O \) directly above the tip of the cone.
2. Let \( P \) be a point in the ground plane. It is required to construct a point \( P' \) in the ground plane so that reflected in the mirror – seen from \( O \) – it appears to be \( P \).
3. In other words, \( P' \) satisfies the following condition:
   - Let \( OP \) intersect the cone at \( U \). A ray from \( P' \) striking the cone at \( U \) is being reflected along \( UO \).
Conical anamorphosis

1. Let \( p \) be the plane through the cone’s axis parallel to the vertical plane. In case \( P \) is in \( p \), the problem is trivial. We are going to make use of this special case to solve the general problem.

2. Rotate the ground plane around the (vertical) axis of the cone till \( P \) coincides with some point \( Q \) in \( p \) and solve the problem \( Q \). This gives \( R \). Apply the inverse rotation to \( R \) to find \( P' \).

Cylindrical anamorphosis

1. An eye point \( O \) and a cylindrical mirror standing on the horizontal (ground) plane are given.

2. Let \( p \) be the polar plane of \( O \) with respect to the cylinder (i.e., the plane determined by the lines of contact of the tangent planes through \( O \)).

3. Because of the eye does not distinguish between points on the same visual ray, we assume that the light seen from \( O \) is coming from points in \( p \).

Definition and examples – cylindrical anamorphosis

Cylindrical anamorphosis

1. So we need to solve the problem: Let \( P \) be a point in \( p \) and let \( C \) be the point where the segment \( OP \) cuts the cylinder. Determine the point \( P' \) in the ground plane so that the ray \( P'C \) reflects along \( CO \).

2. We let \( Q \) be the point of intersection of \( OP \) and the ground plane. The laws of reflection and some elementary geometry show that \( P'Q = \) and \( CO \) make the same angle with the tangent line at \( Q \) to the circular base and that \( CQ^2 = CQ' \).

3. This solves the problem.
Application

} The most popular form of anamorphosis are plane anamorphosis. We can include the entire group of horizontal road signs to this group. Disproportionately stretched signs painted on the street, from the view of the road users take the natural proportions.

Emmanuel Maignan (1642)
“St. Francis of Paola”

Summary

} Anamorphism is used in interior design, for example as a form of drawings on the walls which, depending on the point of the observation, show another image. They also can be seen in the architecture, where the mirror columns are not only the support of architecture, but also an additional element of the design, in which different figures made on the floor specifically for this purpose are reflected.

Emmanuel Maignan. "St. Francis of Paola".
Reference


Drabbe J. Gabriel-Randour. Ch. Descriptive geometry and anamorphosis, (users.skynet.be/mathema/eng.htm).
