Incidence matrix

Let $G$ be an undirected (multi)graph without loops, with $n$ vertices ($x_1, x_2, \ldots, x_n$) and $m$ edges ($e_1, e_2, \ldots, e_m$).
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$$a_{ij} = \begin{cases} 
1 & \text{if } e_j \text{ is incident to } i^{\text{th}} \text{ vertex } x_i, \\
0 & \text{otherwise.}
\end{cases}$$
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1 & \text{if } e_j \text{ is incident to } i - \text{th vertex } x_i, \\
0 & \text{otherwise.} 
\end{cases}$$

$A(G)$ is called the *incidence matrix*. 

```plaintext
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 
\end{pmatrix}
```
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$A(G)$ is called the *incidence matrix*.

$$A(G) = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 \\
\end{bmatrix}$$
Properties of incidence matrix

- each column of $A(G)$ contains exactly two unities (numbers one), because each edge is incident with the exactly two vertices.

Edge $a$ is incident with vertices $x_1$ and $x_2$.

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 \\
\end{bmatrix}
$$
Properties of incidence matrix

- each column of $A(G)$ contains exactly two unities (numbers one), because each edge is incident with the exactly two vertices
- the number of ones in each row is equal to the degree of corresponding vertex

\[
\begin{align*}
\text{deg}(x_2) &= 3 \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 \\
\end{bmatrix}
\end{align*}
\]
Properties of incidence matrix

- each column of $A(G)$ contains exactly two unities (numbers one), because each edge is incident with the exactly two vertices
- the number of ones in each row is equal to the degree of corresponding vertex
- row consisting of zeros represents an isolated vertex
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- the number of ones in each row is equal to the degree of corresponding vertex
- row consisting of zeros represents an isolated vertex
- parallel edges form identical columns

Edges $c$ and $d$ are parallel

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 \\
\end{bmatrix}
\]
From an algorithmic point of view, the matrix of incidence is the worst form of representation of the graph.
Properties of incidence matrix cont.

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1. It requires defining of an array with \( n \cdot m \) entries, most of which is filled with zeros,
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1. it requires defining of an array with $n \cdot m$ entries, most of which is filled with zeros,

2. the answer to the question whether there is an edge connecting given vertices requires $m$ steps (one has to search $m$ columns).
If the graph $G$ has loops, the incidence matrix $A(G)$ can be defined as follows:
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$$a_{ij} = \begin{cases} 
1 & \text{if edge } e_j \text{ is incident with } i-th \text{ vertex } x_i \\
2 & \text{if edge } e_j \text{ is a loop around } x_i, \\
0 & \text{otherwise} 
\end{cases}$$
Let $G$ be an undirected (multi)graph with $n$ vertices $(x_1, x_2, \ldots, x_n)$. 

We define the matrix $A(G) = [a_{ij}]_{n \times n}$ as follows:

- $a_{ij} = \begin{cases} 
\text{the number of edges connecting vertex } x_i \text{ with vertex } x_j \\
0 \text{ if there is not any edge from } x_i \text{ to } x_j 
\end{cases}$

A square matrix defined in the above way is called the adjacency matrix.
Let $G$ be an undirected (multi)graph with $n$ vertices $(x_1, x_2, \ldots, x_n)$.

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We define the matrix $A(G) = [a_{ij}]_{n \times n}$ as follows:

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$$A(G) =$$
Properties of a adjacency matrix

- along the main diagonal all elements are zeros if and only if the graph has no loops.

Edge $f$ is a loop with $x_4$, so $a_{44} = 1$ and $k$ is a loop with $x_2$, so $a_{22} = 1$. 

$$
\begin{bmatrix}
0 & 2 & 0 & 1 & 3 \\
2 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 \\
3 & 0 & 0 & 1 & 0
\end{bmatrix}
$$
Properties of a adjacency matrix

- along the main diagonal all elements are zeros if and only if the graph has no loops.
- if the graph does not have any loop (or on the diagonal it has zeros), the degree of a given vertex is equal to the sum of the elements in the corresponding row or column of matrix $A(G)$

$\text{deg}(x_5) = 4$
**Properties of an adjacency matrix**

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$$\text{deg}(x_5) = 4$$

The adjacency matrix of the graph is:

$$
\begin{bmatrix}
0 & 2 & 0 & 1 & 3 \\
2 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 \\
3 & 0 & 0 & 1 & 0
\end{bmatrix}
$$
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- if the graph does not have any loop (or on the diagonal it has zeros), the degree of a given vertex is equal to the sum of the elements in the corresponding row or column of matrix $A(G)$
- any adjacency matrix of undirected graph is a symmetric matrix

\[
\begin{pmatrix}
0 & 2 & 0 & 1 & 3 \\
2 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 \\
3 & 0 & 0 & 1 & 0
\end{pmatrix}
\]
Advantage

The answer to the question of whether there is an edge from $x_i$ to $x_j$ one gets in one step.
Properties of adjacency matrix

**Advantage**

The answer to the question of whether there is an edge from $x_i$ to $x_j$ one gets in one step.

**Disadvantage**

Regardless of the number of edges in the graph an adjacency matrix stores $n^2$ units of memory.
Properties of adjacency matrix

Theorem

Let $A(G)$ be adjacency matrix of a graph $G$. Then the number of paths of the length $k$ between vertices $x_i$ and $x_j$ is equal to $b_{ij}$, where $B = A^k$. 
Example

Paths of the length 2 we obtain from $A^2$.
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$$B = A^2 = \begin{bmatrix}
0 & 2 & 0 & 1 & 3 \\
2 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 \\
3 & 0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
0 & 2 & 0 & 1 & 3 \\
2 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 \\
3 & 0 & 0 & 1 & 0
\end{bmatrix} = \begin{bmatrix}
14 & 3 & 1 & 6 & 1 \\
3 & 6 & 1 & 4 & 7 \\
1 & 1 & 1 & 1 & 1 \\
6 & 4 & 1 & 5 & 4 \\
1 & 7 & 1 & 4 & 10
\end{bmatrix}$$
Example

For instance

\[ b_{42} = a_{41}a_{12} + a_{42}a_{22} + a_{43}a_{32} + a_{44}a_{42} + a_{45}a_{52} = 1 \cdot 2 + 1 \cdot 1 + 1 \cdot 0 + 1 \cdot 1 + 1 \cdot 0 = 4 \]

These are paths gh, gi, jk, fj

Paths of the length 2 we obtain from \( A^2 \).
Representations of undirected graphs

Example

We get

\[ A^3 = \begin{bmatrix} 15 & 37 & 6 & 25 & 48 \\ 37 & 16 & 4 & 21 & 13 \\ 6 & 4 & 1 & 5 & 4 \\ 25 & 21 & 5 & 20 & 23 \\ 48 & 13 & 4 & 23 & 7 \end{bmatrix} \]

\[ A^4 = \begin{bmatrix} 243 & 92 & 25 & 131 & 70 \\ 92 & 111 & 21 & 91 & 132 \\ 25 & 21 & 5 & 20 & 23 \\ 131 & 91 & 20 & 94 & 95 \\ 70 & 132 & 23 & 95 & 167 \end{bmatrix} \]
Consider a (multi)graph $G$ with $m$ edges.

The list of edges of $G$ is implemented with the aid of two lists

$$F = (f_1, f_2, \ldots, f_m),$$
$$H = (h_1, h_2, \ldots, h_m).$$

where $\{f_i, h_i\}$ represents an edge of $G$ (which links vertices $f_i$ and $g_i$).
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where $\{f_i, h_i\}$ represents an edge of $G$ (which links vertices $f_i$ and $g_i$).

We have

$$F = (1, 1, 1, 1, 2, 2, 3, 3, 3),$$
$$H = (1, 2, 2, 4, 3, 4, 4, 4, 5).$$
Properties of lists of edges

Advantage

To represent the graph with $m$ edges we need $2m$ memory units (it is convenient for rare graphs where the number of edges is much less than the number of vertices).
Properties of lists of edges

Advantage
To represent the graph with $m$ edges we need $2m$ memory units (it is convenient for rare graphs where the number of edges is much less than the number of vertices).

Disadvantage
To obtain a set of vertices, which are linked with a given vertex we need, in the worst case, $m$ steps.
The adjacency list is built for each vertex of the (multi)graph.
Adjacency lists

The adjacency list is built for each vertex of the (multi)graph. For a given vertex $x_i \in V$ we define the list of such vertices $u$, that $\{x_i, u\}$ is an edge in the graph $G$. 
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Adjacency lists:

1:
2:
3:
4:
5:
Properties of adjacency lists

- any adjacency list consists of vectors of lists
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- each edge in any undirected graph is stored in two different places, if we consider the edge \( \{u, w\} \), \( w \) is the vertex on the list of vertex \( v \) and vice versa (this observation does not apply to the loop)
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Advantages

- the number of memory units needed to store the graph with \( n \) vertices and \( m \) edges is, in the worst case, proportional to \( m + n \) (generally it is enough to use \( 2m \) units of memory),
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Advantages

- the number of memory units needed to store the graph with \( n \) vertices and \( m \) edges is, in the worst case, proportional to \( m + n \) (generally it is enough to use \( 2m \) units of memory),
- testing of the existence of a single edge needs the time proportional to \( n \),
- adjacency list allows to track all the links form a given vertex.
Let $G$ be a digraph without loops with vertices $x_1, x_2, ..., x_n$ and edges $e_1, e_2, ..., e_m$. 

An incidence matrix $A(G) = [a_{ij}]_{\times_n \times_m}$ for $i = 1, ..., n$, $j = 1, ..., m$ is defined as follows:

- $a_{ij} = 1$ if $x_i$ is the tail of $e_j$,
- $a_{ij} = -1$ if $x_i$ is the head of $e_j$,
- $a_{ij} = 0$ otherwise.
Incidence matrix

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1 & \text{if } x_i \text{ is the tail of } e_j \\ 
-1 & \text{if } x_i \text{ is the head of } e_j \\ 
0 & \text{otherwise}
\end{cases}$$

$$A(G) = \begin{bmatrix}
1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & -1 & -1 & -1 & 0 \\
0 & -1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$
Adjacency matrix for a directed graph is defined identically as for the undirected graph. Recall that

\[ a_{ij} = \begin{cases} 
\text{the number of edges connecting vertex } x_i \text{ with vertex } x_j \\ 
0 \text{ if there is not any edge from } x_i \text{ to } x_j 
\end{cases} \]
Similarly as for undirected graphs, successive powers of digraph adjacency matrix contains a number of paths between vertices.

Adjacency matrix is of the form

\[
A = \begin{bmatrix}
0 & 0 & 0 & 1 \\
2 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]
Similarly as for undirected graphs, successive powers of digraph adjacency matrix contains a number of paths between vertices.

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A = \begin{bmatrix}
0 & 0 & 0 & 1 \\
2 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

Then

\[
A^2 = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \quad A^3 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \quad A^4 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
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A = \begin{bmatrix}
0 & 0 & 0 & 1 \\
2 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\]

There are 4 paths of the length of 2:

\[
a_{13}^2 = 1 - \text{a path } ca \\
a_{23}^2 = 1 - \text{a path } ba \\
a_{24}^2 = 2 - \text{two paths } dc \text{ and } ec
\]

Then

\[
A^2 = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
A^3 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
A^4 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
Similarly as for undirected graphs, successive powers of digraph adjacency matrix contains a number of paths between vertices.

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0 & 0 & 0 & 1 \\
2 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

There are 2 paths of the length of 3:

\[a_{23}^3 = 2 - \text{two paths } dca \text{ and } eca\]

Then

\[
A^2 = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
A^3 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
A^4 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
Similarly as for undirected graphs, successive powers of digraph adjacency matrix contains a number of paths between vertices.

Adjacency matrix is of the form

\[
A = \begin{bmatrix}
0 & 0 & 0 & 1 \\
2 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

In the digraph there is no any path of the length of 4 or more

Then

\[
A^2 = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
A^3 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
A^4 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
Similarly as for a undirected graph, the adjacency list is built for every vertex.
Similarly as for an undirected graph, the adjacency list is built for every vertex. For vertex $x_i \in V$, it consists of such vertices $u$, such that $(x_i, u)$ is an edge in $G$. 
In the adjacency list for digraphs every edge is stored only once.
Representations of undirected graphs

Representations of directed graphs

Matrix representation

Adjacency list

In the adjacency list for digraphs every edge is stored only once.

```
x : y
y : z
z : x, u
u : x, y
```
In the adjacency list for digraphs every edge is stored only once.
Thank you for your attention!!!