Mathematical League of University of Lodz

Series I 22/23

For every exercise you can get max. 10. p. Solutions should be delivered on paper (every task on the separate piece of paper) to the room B207 or electronically on the address: piotr.nowakowski@wmii.uni.lodz.pl. Deadline: 16.11.2022.

Zadanie 1. *Prove that there does not exist an arithmetical sequence such that its three consecutive terms are also different terms of the sequence* (2^n).

Zadanie 2. *What is the greatest and the smallest number (not starting from* 0*), consisting of all digits from* 0 *to* 9 *(where every digit appears exactly once) and is divisible by* 11*?*

Zadanie 3. *Let* $M \subset \mathbb{N}$ *be a set such that for any* $n, m \in M$

$$n > m \Rightarrow n - m \geq \frac{nm}{25}.$$

What is the maximal number of elements of the set M*?*

Zadanie 4. *For* $n \in \mathbb{N}$ *solve the following equation in integers:*

$$\left(2 - \frac{1}{x_1}\right)\left(2 - \frac{1}{x_2}\right)\ldots\left(2 - \frac{1}{x_n}\right) = 3.$$

Zadanie 5. *Fix* $n \in \mathbb{N} \setminus \{1\}$*. Two players play the following game. Starting from* $k = 2$*, each player has two possible moves: he can replace* k *by* $k+1$ *or by* $2k$*. The player who is forced to choose a number greater than* n *loses. For each* n *which player (the first one or the second one) has a winning strategy, that is, a strategy which use will ensure the win no matter what the other player will do?*